Comment on "Perturbations in the Ionosphere Caused by a Moving Body"

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THE present author would like to comment on Ref. 1, in which the problem concerns a spherically shaped body which is penetrating a region of low density. In part one, titled "Density of Neutral Particles," it is desired to determine how particles fill the region behind the body when all collisions are neglected. It is argued that the body cross section in the plane orthogonal to the incident flux is the geometrical characteristic of primary importance. Thus, a disk of radius R_0 (lying in the plane $z = 0$) is substituted for the sphere. The procedure is to find a steady state solution for the distribution function f by solving the Boltzmann equation in a coordinate system affixed to the disk (Fig. 1).

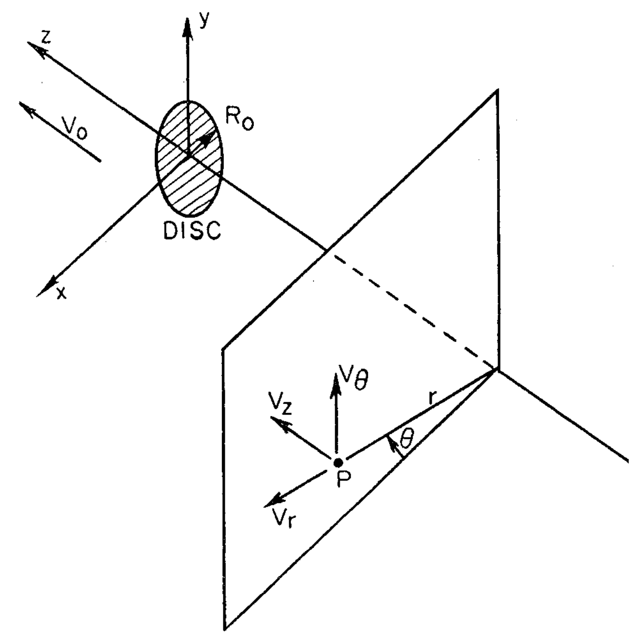

Fig. 1 Geometry associated with the moving disk

However the Boltzmann equation (without the collision term) in cylindrical coordinates r,θ,z, is incorrectly given as (Eq. 7):

$$V_r \frac{\partial f}{\partial r} + (V_z + V_0)\frac{\partial f}{\partial z} = 0$$

If classical laws of motion are assumed correct, all coordinate systems translating with arbitrary velocities are indistinguishable. Inclusion in the Boltzmann equation of the term $V_0(\partial f/\partial z)$ would violate this premise. Also, a little care must be exercised when the Boltzmann equation is written in a coordinate system which has curvature (any system other than Cartesian). Even when no external forces are present, coordinate system curvature will introduce "apparent" forces. More specifically, the term

$$(d\mathbf{v}/dt)\cdot\nabla_{\mathbf{v}} f$$

[or $(1/M)\nabla_{\mathrm{r}}\, u\cdot\nabla_{\mathbf{v}} f$ as in Eq. (4)] is not zero. The additional force terms in cylindrical coordinates may be determined by expressing V_r, V_θ (see Fig. 1) in terms of V_x, V_y, and θ; and taking time derivatives of V_r, V_θ while holding V_x, V_y constant. The correct equation for f in a cylindrical coordinate system with axial symmetry, and moving with the disk, is

$$\frac{\partial f}{\partial t} + V_r\frac{\partial f}{\partial r} + V_z\frac{\partial f}{\partial z} + \frac{V_\theta^2}{r}\frac{\partial f}{\partial V_r} - \frac{V_r V_\theta}{r}\frac{\partial f}{\partial V_\theta} = 0$$

Since the flow is steady while riding with the disk, the first term can be neglected.

If the initial value of f is [Eq. (6)]:

$$f_{\text{at } z=0} = n_{m_0}\left(\frac{M}{2\pi kT}\right)^{3/2} \times H(\sqrt{r^2} - R_0)\, e^{-(M/2kT)[V_x^2 + V_y^2 + (V_z + V_0)^2]}$$

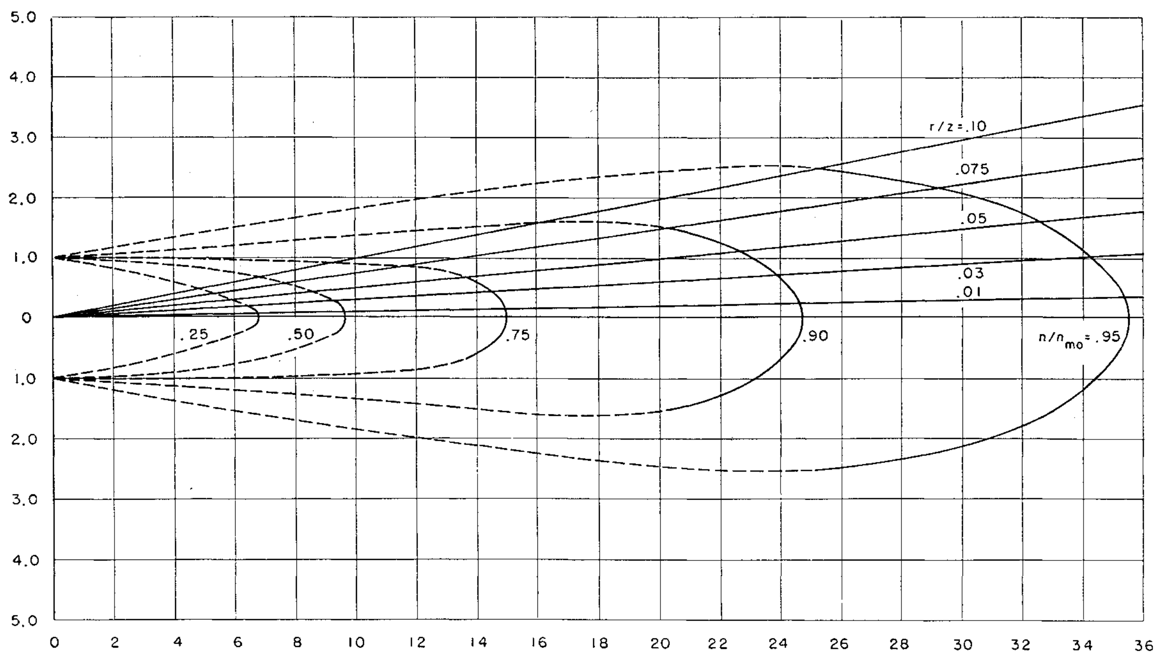

Fig. 2 Contours of constant number density for $V_0 = 8\sqrt{2kT/M}$

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where

R_0 = disk radius
$H(x)$ = step function
V_0 = velocity of disk

then the proper solution for f in the region $z < 0$ is

$$f = n_{m_0}\left(\frac{M}{2\pi kT}\right)^{3/2} \times H\left(\sqrt{\left(x - \frac{V_x z}{V_z}\right)^2 + \left(y - \frac{V_y z}{V_z}\right)^2} - R_0\right) \times e^{-(M/2kT)[V_x^2 + V_y^2 + (V_z + V_0)^2]}$$

(It is more convenient to work in a Cartesian system, because the solution is not simplified by the use of cylindrical coordinates.)

The number density n is expressed as an integral over velocity space:

The integration can be done easily for $r/z = 0$, but an approximate technique was used to obtain simple expressions for the number density near the axis of symmetry. The results are shown in Fig. 2 for a value of V_0 equal to $8\sqrt{2kT/M}$. Here r and z are normalized by the disc radius and n_{m0} is the number density in the plane $z = 0$. The dotted portions of the curves represent regions where calculations were not made. However, by using the known initial conditions and the exact solution along the axis of symmetry, the dotted portions can be sketched with reasonable accuracy.

The reasonably close agreement between the results represented in Fig. 2 and those presented in Ref. 1 is difficult to explain. It appears that other errors have been made in the evaluation of the number density in Ref. 1 which discourage any logical comparison of similarities.

Reference

[1] Gurevich, A. V., "Perturbations in the ionosphere caused by a moving body," ARS J. 32, 1161–1167 (1962).

$$n = n_{m_0}\left(\frac{M}{2\pi kT}\right)^{3/2} \iiint H\left(\sqrt{\left(x - \frac{V_x z}{V_z}\right)^2 + \left(y - \frac{V_y z}{V_z}\right)^2} - R_0\right) e^{-(M/2kT)[V_x^2 + V_y^2 + (V_z + V_0)^2]}\, dV_x dV_y dV_z$$

Digest of Translated Russian Literature

The following abstracts have been selected by the Editor from translated Russian journals supplied by the indicated societies and organizations, whose cooperation is gratefully acknowledged. Information concerning subscriptions to the publications may be obtained from these societies and organizations. Note: Volumes and numbers given are those of the English translations, not of the original Russian.

Soviet Physics—Technical Physics (*Zhurnal Tekhnicheskoi Fiziki*). Published by American Institute of Physics, New York

Volume 6, Number 5, November 1961

Production of Negative Hydrogen Ions in the Passage of Protons through Gas Targets, Yu. M. Khirnyi, pp. 427–432.

Results are given of an investigation of the content of H_1^-, H_1^+, and H_1^0 in equilibrium beams obtained in the passage of protons through gas targets of H_2, He, Ne, Ar, CO_2, and C_3H_8; data on the scattering of hydrogen beams on these targets are also given.

In sources of negative hydrogen ions one usually makes use of charge exchange in the passage of positive hydrogen ions through matter. The usual charge-exchange target is either a point hydrogen target or a supersonic mercury jet. Since the H_1^-

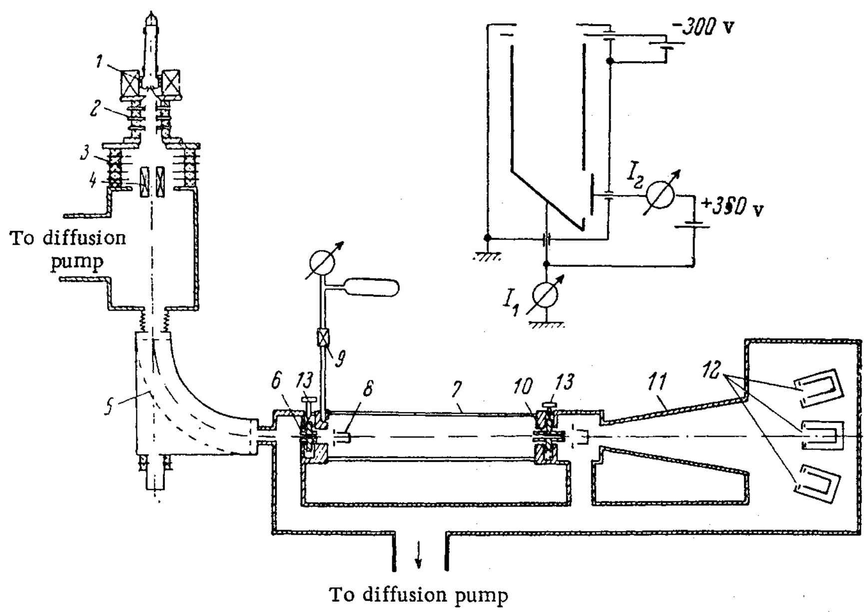

Fig. 1.